國立臺灣師範大學 111 學年度學士班二年級轉學生招生考試試題

科目：微積分　　　　　　　　　　適用學系（組）：數學系

注意：1. 本試題共 1 頁，請依序在答案本上作答，並標明題號，不必抄題。
　　　2. 答案必須寫在答案本上之指定作答區內，否則依規定予以扣分。

一、 填充題 (答案本上只寫答案，不需要寫計算過程，請標明題號)

1. $y = (\ln x)^{\ln x}$, find $\dfrac{dy}{dx}$. (8 points)

2. Find the limit: $\displaystyle\lim_{x \to 1} \left(\dfrac{11}{1 - x^{11}} - \dfrac{5}{1 - x^5} \right)$. (8 points)

3. Find the arc length of the graph of the function $x = \dfrac{1}{3}\sqrt{y}(y - 3)$ over the interval $1 \le y \le 4$. (8 points)

4. Find the volume of the solid generated by revolving the plane region bounded by the graph of $(y - 2)^2 = 4 - x$ and y-axis about the line x-axis. (8 points)

5. Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}(x - 4)^n}{n \cdot 3^n}$. (8 points)

6. Find the area of the region common to the two regions bounded by the curves $r = -3\cos\theta$ and $r = 1 - \cos\theta$. (8 points)

7. Find relative maximum of the function $f(x, y) = -x^3 + 4xy - 2y^2 + 1$. (8 points)

8. Evaluate $\displaystyle\int_0^\infty \int_0^\infty e^{-(\frac{x^2 + 3y^2}{4})} \, dx dy$. (8 points)

二、 計算證明題 (請在答案本上寫出計算過程和答案，沒有過程不予計分)

1. Explain the theory in (a),(b) without proof, then prove (c).

   (a) The Intermediate Value Theorem of the continuous function. (5 points)

   (b) The Mean Value Theory of the continuous function (differential form). (5 points)

   (c) Prove that $|\cos a - \cos b| \le |a - b|$ for all real number a, b. (8 points)

2. (a) Find the limit: $\displaystyle\lim_{(x,y) \to (0,0)} \dfrac{xy^2}{2x^2 + 3y^4}$. (8 points)

   (b) Use triple integral and change of variables to find the volume of the solid region $\dfrac{x^2}{9} + \dfrac{y^2}{4} + z^2 \le 1$. (10 points)