國立臺灣師範大學 111 學年度轉系考試試題

科目：微積分　　適用學系所：數學系

注意：1. 本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2. 答案必須寫在指定作答區內，否則依規定扣分。

1. Evaluate the following limits. (8 points each)

   (1) $\lim_{x\to 0} \dfrac{\tan(5x)}{\sin(2x)\cos(3x)}$　　(2) $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$

   (3) $\lim_{n\to\infty} \dfrac{\pi}{3n}\left(\tan\dfrac{\pi}{3n} + \tan\dfrac{2\pi}{3n} + \cdots + \tan\dfrac{\pi}{3}\right)$

2. Evaluate f_x and f_y of the function $f(x,y) = \arctan\dfrac{y}{x}$ at the point $(2,2)$. (10 point)

3. Find the equation of the tangent lines at the point where the parametric curve $x = t^3 - 6t$, $y = t^2$ cross itself (自交). (10 point)

4. Evaluate the following integrals: (8 points each)

   (1) $\displaystyle\int_0^1 x\ln x\ dx$　　(2) $\displaystyle\int_0^{\pi/4} \tan^5 x\ dx$　　(3) $\displaystyle\int_0^1\int_0^{1-x} \sqrt{x+y}(y-2x)^2\ dydx$

5. Find the area of the region bounded by the circle $r = 2\sin\theta$ for $\pi/4 \le \theta \le \pi/2$. (10 points)

6. Find all the asymptotes of the function $f(x) = \dfrac{2x^3}{x^2-1}$. (12 points)

7. If $\displaystyle\int_0^{x^{10}} f(2t-\sqrt{t}+7)\ dt = e^x$, find $f(2023)$. (10 points)