# 國立臺灣師範大學 110 學年度轉系考試試題

科目：微積分　　適用學系所：數學系

注意：1. 本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2. 答案必須寫在指定作答區內，否則依規定扣分。

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1. Evaluate the limit $\lim\limits_{x\to 1^+} \dfrac{\int_3^{3x^2} \sqrt{\sec\theta}\, d\theta}{x-1}$. (8 points)

2. Evaluate the following integrals: (8 points each)

   (1) $\displaystyle\int \frac{\sqrt{x^2-25}}{x}\, dx$ (2) $\displaystyle\int_0^{\infty} x^2 e^{-x}\, dx$ (3) $\displaystyle\int_0^2 \int_{\frac{1}{2}x^2}^2 \sqrt{y}\cos y\, dydx$

   (4) $\displaystyle\iint_R \ln(x^2+y^2)\, dA$, area $R: 1 \le \sqrt{x^2+y^2} \le 2$.

3. Find the length of the curve $y = \ln(\sin x)$ for the given interval $\left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$. (10 points)

4. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+2}{n(n+1)}$ is absolutely convergent or conditionally convergent. (10 point)

5. Find the slope and concavity of the curve $x = \cos^4\theta,\ y = \sin^4\theta$ at $\theta = -\dfrac{\pi}{3}$. (10 points)

6. Find the minimum distance from the surface $z = \sqrt{x^2+y^2}$ to the point $(4,0,0)$. (10 points)

7. Find the volume of the solid generated by revolving the region bounded by the graph of curve $y = x\sqrt{x+1}$ and $y = 0$ about the $y$-axis. (10 points)

8. Giving function $f(x,y) = \dfrac{x^2+y^2}{xy}$ if $xy \neq 0$ and $f(x,y) = 0$ if $xy = 0$. Determine whether the directional derivative $D_{\mathbf{u}} f(0,0)$ exists or not, where the direction $\mathbf{u} = \dfrac{1}{2}\mathbf{i} + \dfrac{\sqrt{3}}{2}\mathbf{j}$. (10 points)