博士班資格考試筆試各科目考試範圍與參考書目

Guidelines of Qualify Exams for admission to official candidacy for the PhD degree

實變分析 (Real Analysis)

考試範圍 (Scope)：

(打*不考定理證明, the formal proof will not be tested for those theorem starred)

1. Functions of Bounded Variational

   Jordan theorem, Riemann-Stieltjes integral.

2.. Lebesgue Measure and Outer Measure

   Algebra, σ -algebra, Borel sets, Measure spaces, Littlewood 3 principles, Cantor sets, Cantor-Lebesgue functions, Caratheodory measurable sets, Steinhaus theorem, Vitali nonmeasurable sets.

3. Lebesgue Measurable Functions

   Lusin theorem, Egorov theorem, Convergence a.e. (in measure, in L^P)

4. Lebesgue Integral

   Convergence theorem (MCT, LDCT, BCT, UCT), Fatou lemma, Tchebyshey inequality, Relation between Riemann-Stieltjes integral and Lebesgue integral.

5. Repeated Integration

   Fubini theorem, Tonelli theorem, Convolution.

6. Differentiation

   Indefinite integral, Absolute continuous, Vitali covering lemma*, Lebesgue differentiation theorem*, Hardy-Littlewood theorem*, Monotone functions, Convex functions.

7. L^P Classes

   Essential supremum, Normed linear spaces, Banach Spaces, L^P spaces, ℓ^P spaces*, Separable spaces, Dual spaces, Holder inequality, Minkowski inequality, Hahn-Banach theorem, Parseval formula, Bessel inequality, Complete orthonormal system, Riesz-Fischer theorem.

8. Abstract Measure Theory

Signed measure, Additive set measure, Radon-Nikodym theorem.

参考書目 **(References):**

R. L. Wheeden and A. Zygmund, Measure and Integral, An Introduction to Real Analysis (Chapter 2,3,4,5,6,7,8,10)