國立臺灣師範大學 108 學年度學士班二年級轉學生招生考試試題

科目：微積分　　適用學系（組）：數學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

一、填充題 (答案本上只寫答案，不需要寫計算過程，請標明題號)

1. (20 分) Find the following definite integrals.
   (a) $\int_0^1 e^x \sin x \, dx$,
   (b) $\int_0^1 \ln(1+x^2) \, dx$.

2. (8 分) Find an equation of the tangent line to the graph $x^2 + xy + y^2 = 9$ at the given point $(x, y) = (3, 0)$.

3. (8 分) Find the value of c that makes the following function f continuous at $x = 0$.

$$f(x) = \begin{cases} (2x - 2\sin x)/(2x^3), & \text{if } x \neq 0, \\ c, & \text{if } x = 0. \end{cases} \tag{1}$$

4. (8 分) Find the derivative $F'(0)$, where

$$F(x) = \int_{2x}^{x^3} \cos(t^2) \, dt.$$

5. (8 分) Find the minimum value of

$$f(x, y, z) = 2x^2 + y^2 + 3z^2$$

subject to the constraint $2x - 2y - 3z = 45$.

6. (8 分) Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where C is a piecewise smooth curve from $(-1, 4)$ to $(1, 2)$, the symbol " $\cdot$ " is the dot product, **r** is a parametrization of the curve C, and

$$\mathbf{F}(x, y) = (3x^2y, x^3 - y).$$

7. (8 分) Find the surface area of the portion of the plane $z = 3 - x - y$ that lies above the disc $x^2 + y^2 \leq 1$ in the first quadrant.

8. (8 分) Evaluate the line integral of f with respect to arc length parameter s

$$\int_C f(x, y, z) ds,$$

where $f(x, y, z) = (2/\sqrt{3})x + 1$, and C is the space curve represented by

$$(x(t), y(t), z(t)) = \left(\sqrt{3}t, \frac{2}{3}t^{\frac{3}{2}}, \frac{1}{2}t^2\right), \ 0 \leq t \leq 2.$$

9. (10 分) Evaluate the iterated integral

$$\int_0^1 \int_{y/2}^{1/2} e^{-x^2} dx dy.$$

二、計算證明題 (請在答案本上寫出計算過程和答案，沒有過程不予計分)

1. (14 分) Let f be a continuous function on $[a, b]$. Prove that

$$\lim_{h \to 0} \frac{1}{h} \int_a^x [f(t+h) - f(t)] dt = f(x) - f(a), \ (a < x < b).$$