1. Find the following integrals. (14 pts)

   (a) $\int \sec^3\theta\, d\theta$ (b) $\int \dfrac{e^{2z}}{e^{2z}+3e^z+2}\, dz$

2. Find the tangent to the curve $x^y = y^x$ at the point $(1,1)$. (6 pts)

3. Let $f$ be a real-valued function defined by

$$f(x) = \begin{cases} x\sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0. \end{cases}$$

   Is $f$ differentiable at $x = 0$? Give your reasons. (10 pts)

4. If two real-valued functions $f$ and $g$ are defined by

$$f(x) = \int_1^x \frac{dt}{\sqrt{t^2+1}}, \quad g(x) = \ln(x-1)$$

   for all $x > 1$, find the horizontal asymptote for $f(x)/g(x)$. (14 pts)

5. Prove that $x^3 + 3x^2 + 3x - 1 = 0$ has *exactly* one real root. (6 pts)