科目：微積分　　　　　　　　適用學系（組）：數學系

1. (10 points) Find the limit $\lim_{h\to 0} \dfrac{1-\cos 3h}{\cos^2 5h - 1}$.

2. (10 points) Is the function

$$f(x) = \begin{cases} x\sin(\dfrac{1}{x}) & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

   differentiable at $x = 0$?

3. (10 points) Evaluate the integral $\int \dfrac{\sec(\sqrt{x})}{\sqrt{x}} dx$.

4. (10 points) Does the improper integral $\int_0^\infty \dfrac{1}{(x^2+1)^2} dx$ converge?

5. A surface σ is the part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 2$.

   (a) (10 points) Write down the equation of the tangent plane of σ at the point $(3/2, 3/2, 3/\sqrt{2})$.

   (b) (15 points) Evaluate the surface integral $\int\int_\sigma y^2 z^2 dS$.

6. A solid hemisphere H is enclosed by $z = \sqrt{a^2 - x^2 - y^2}$ and $z = 0$.

   (a) (10 points) Find the volume of H.

   (b) (25 points) Suppose a vector field $\vec{F}(x,y,z) = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ is defined over the surface σ of H. Let $\vec{n}$ denote the outward unit normal vector of σ. Evaluate the integral $\int\int_\sigma \vec{F} \cdot \vec{n}\, dS$.